Global Derivatives Trading & Risk Management

Ogee Group LLC

A Primer on Correlation Trading via Equity Derivatives

By Sebastien Bossu for the Global Derivatives Live blog, April 2016.

This blog post is written in connection with Sebastien Bossu's upcoming Global Derivatives conference talk "[Advances in Equity Correlation: Some Fundamental Cutting-Edge Results]" on Thursday 12th May at 12:45 – 1:15pm.

In previous editions of the Global Derivatives conference, I would spend some 15 minutes explaining variance swaps, vanilla and variance dispersion trading and correlation swaps to make sure everyone was on the same page and to motivate the rest of my talk. This year, I plan to jump directly into the more technical aspects, which is why I wrote this primer. For more technical details, see also my book ADVANCED EQUITY DERIVATIVES: VOLATILITY & CORRELATION (Wiley, 2014).

1. Variance Swaps/Futures

Variance swaps are OTC exotic derivatives and were developed in the 1990s by researchers and practitioners alike. They pay off the realized variance of an asset—say, the S&P 500 index—observed between the trade date and a given maturity date, minus a fixed price. Here, "realized variance" is the square of the annualized standard deviation of S&P 500 daily returns, and the fixed price can be seen as "implied variance"—the market expectation of realized variance:

VarSwap Payoff = Realized Variance – Implied Variance

If this already sounds unappealing, contrived, unintuitive and awfully academic to you, you're completely right: practitioners understand volatility, not variance, and, in fact, there are also OTC volatility swaps out there that work like VIX futures, except that they deliver realized volatility rather than the VIX.

So why bother with variance swaps, or even volatility swaps, at all when there is a very liquid VIX futures market? It turns out that only variance can be hedged cost-effectively with a static portfolio of puts and calls; in contrast, volatility swaps and VIX futures require the dynamic trading of options, which isn't practical. As a result, the variance swap price is model-independent, whereas pricing volatility swaps or VIX futures is model-dependent.

If you still think that variance swaps are merely conceptual, check out your Bloomberg screens. You'll see that all major option brokers have variance swap pages. There is even an exchange-traded version called variance futures[1].

Let me now introduce some mathematical notations:

$$\text{VarSwap Payoff}_T = \sigma^2 - \sigma^{\star 2}$$

[1] At present variance futures very rarely trade and unsophisticated brokers will rebuff at their idiosyncrasies, but there's a market-maker and I've successfully traded them myself on a monthly basis for some period of time.

where 0 is the trade date, T is the maturity date, σ is the realized volatility observed between 0 and T, and $\sigma^\star$ is known as the strike of a variance swap trading at time 0 with a maturity date T. Here σ and $\sigma^\star$ are both expressed in volatility points (e.g., 20 for 20%).

For example, the strike of a one-year variance swap on the S&P 500 could be 20 volatility points (corresponding to and implied variance of $20^2 = \$400$), and if the realized volatility of the S&P 500 after one year is 30%, the payoff would be $30^2 - 20^2 = \$500$.

2. Correlation Trading

Two main motivations exist for trading correlation: hedging (mostly on the sell side) and alpha (mostly on the buy side). In equity derivatives, exotic trading desks tend to be short correlation[2] as they satisfy investors' demand for structured products that are predominantly long correlation. As a result, implied correlation tends to consistently trade at a premium versus realized correlation—the former being the level of correlation priced into multi-asset options and the latter being the actual level of statistical correlation observed between the trade date and the maturity date.

This, in turn, generates attractive alpha for the happy few who can successfully capture the gap between implied and realized correlation. Figure 1 below shows this gap between 2002 and 2013 for the portfolio of the 50 constituent stocks of the EuroStoxx index weighted by market capitalizations. We can see that anyone who is able to capture the gap would almost never have made a loss.

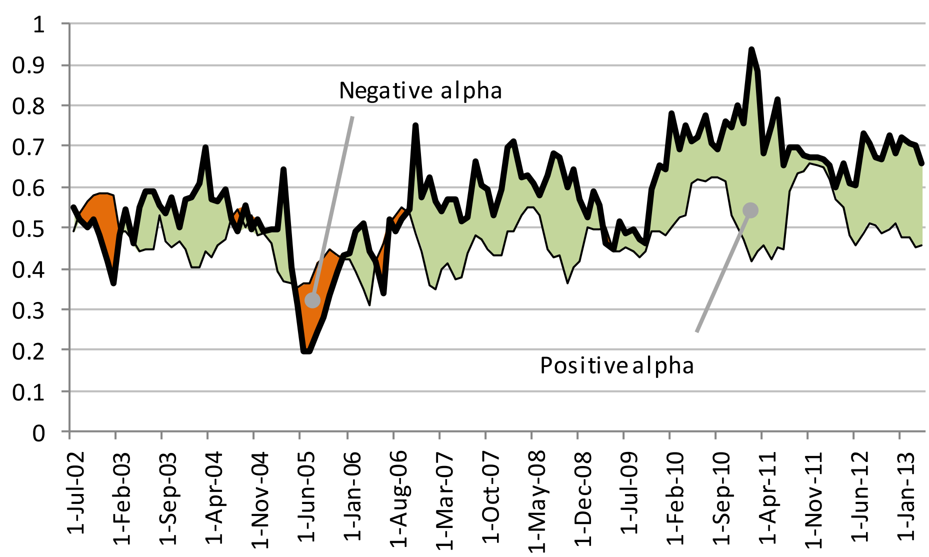

Figure 1: 6-month variance-implied correlation (bold line) and realized correlation 6 months later (thin line) for the EuroStoxx 50 index. (Data sources: OptionMetrics, Bloomberg. Reprinted from my book ADVANCED EQUITY DERIVATIVES: VOLATILITY & CORRELATION, Wiley, 2014.)

[2] A short correlation exposure means that an increase in correlation (implied or realized) loses money. Likewise, a long volatility exposure means that an increase in volatility makes money, and so on.

Of course, trading correlation is not easy. If it was, everyone would do it, and the alpha would disappear pretty quickly. A few months ago, I met with an executive at a multi-billion-dollar hedge fund with a fancy office near Wall Street specializing in option arbitrage, and he told me that their attempt at correlation arbitrage had not been very successful. Whether this was because they could not get the right execution talent for this type of trading or some other mystery that only a multi-billion-dollar organization can afford to leave unsolved, I can merely speculate. That being said, I have heard from different sources that a handful of sophisticated pension funds have become pretty good at it.

I will now review the three main instruments one can use to trade correlation: vanilla dispersions, variance dispersions and correlation swaps.

2.1. Vanilla Dispersions

Vanilla dispersions were the first correlation trades to appear. In equity derivatives, they are structured as follows:

Long Vanilla Dispersion ≡ Long Basket Straddle and Short Single-Stock Straddles

The long basket straddle leg is, broadly speaking, long volatility and long correlation; in contrast, the short single-stock straddles leg is short volatility and correlation-neutral. In principle, by weighting legs appropriately, one can obtain a net exposure that is long correlation and volatility neutral, and it turns out that such a trade has zero cost.

From a trading perspective, vanilla dispersion trades are attractive because they tend to be liquid, cost-effective and customizable. However, their major disadvantage is that they need to be delta-hedged, and the daily delta-hedging P&L will depend on the gammas of the basket and single stock options. The final P&L is then only very loosely connected to correlation.

2.2. Variance Dispersions

Thanks to the expansion of variance swap markets, in the mid-2000s it became possible to trade variance dispersion trades:

Long Variance Dispersion ≡ Long Basket Variance Swap and Short Single-Stock Variance Swaps

Again, the basket variance swap leg is long volatility and long correlation, whereas the short single-stock variance swaps leg is short volatility and correlation neutral. By weighting legs appropriately, one can obtain a net exposure that is long correlation yet volatility-neutral, and it turns out that such a trade has zero cost (i.e., the net fixed cash flow resulting from variance swap strikes is zero).

With mathematical notations, we can write the payoff of a variance dispersion trade as:

$$\text{VarDisp Payoff}_T = \sigma_{\text{Basket}}^2 - \beta \times \sum_{i=1}^{n} w_i \sigma_i^2$$

where σ's are realized volatilities, β is the leg ratio, n is the number of stocks in the basket and w's are stock basket weights. Note that $\sum_{i=1}^{n} w_i \sigma_i^2$ is the weighted average of single-stock variance.

It is easy to show that the zero-cost leg ratio is $\beta = \sigma^{\star 2}_{\text{Basket}} / \sum_{i=1}^{n} w_i \sigma_i^{\star 2}$ and that it is a number ranging between 0 and 1. For example, a beta value of 0.6 means that the quantity N of basket variance swaps to buy must be matched by a smaller quantity $0.6 \times N$ of single-stock variance swaps to sell.

It turns out that β can be seen as a particular measure of implied correlation—the implicit level of average pairwise correlation priced into the basket variance swap—while basket realized variance σ^2_{Basket} can be decomposed as the product of average single-stock realized variance and realized correlation:

$$\sigma^2_{\text{Basket}} = \rho \times \sum_{i=1}^{n} w_i \sigma_i^2$$

This equation is a variant of the proxy formula I discovered in January 2004—only 6 months after I started my banking career as an exotic derivatives analyst at J.P. Morgan in London—whereby:

$$\text{Basket Volatility} \approx \text{Average Volatility} \times \sqrt{\text{Correlation}}$$

Here, to obtain equality, we must measure correlation as $\rho = \sigma^2_{\text{Basket}} / \sum_{i=1}^{n} w_i \sigma_i^2$, whereas the proxy formula uses the more conventional measure $\bar{\rho} = \sum_{i<j} w_i w_j \rho_{i,j} / \sum_{i<j} w_i w_j$. I will explain the proxy formula and how β, ρ and $\bar{\rho}$ relate to correlation in more detail during my talk.

Putting everything together and writing $\rho^\star$ instead of β as a more standard notation for implied correlation, the P&L of a zero-cost variance dispersion trade boils down to:

$$\text{Zero-Cost VarDisp P\&L}_T = (\rho - \rho^\star) \times \sum_{i=1}^{n} w_i \sigma_i^2$$

In other words, the P&L on a zero-cost variance dispersion is simply the spread between realized and implied correlation, multiplied by the average realized single-stock variance. This is already remarkable because, contrary to vanilla dispersions, here, the P&L is directly connected to the spread between realized and implied correlation, which allows for the efficient capturing of the alpha discussed earlier.

As always, the devil is in the details, and there are many practical pitfalls involved in properly executing variance dispersion trades. Additionally, after the 2008 crisis, the liquidity in single-stock variance swaps dried up, which means that easy access to variance dispersions is never guaranteed[3].

2.3. Correlation Swaps

Around 2000, the first correlation swaps were pushed by equity derivatives exotic trading desks looking to offload their short correlation exposure onto sophisticated clients lured in by the prospect of high alpha. Correlation swaps are multi-asset derivatives that pay off the realized correlation across the underlying assets, minus a fixed strike price. Here, "realized correlation" is defined as the average realized correlation coefficient between the daily returns of any stock pair, as observed between the trade date and the maturity date, and the fixed strike price can be seen as yet another measure of "implied correlation," this time as the market expectation of realized correlation:

[3] Note, however, that as long as there is a reasonably liquid option market on each single stock, it is possible to synthesize a variance swap by buying the replicating portfolio of calls and puts.

CorrelSwap Payoff = Realized Correlation – Implied Correlation

Using mathematical notations:

$$\text{CorrelSwap Payoff}_T = \bar{\rho} - \rho^{\star}$$

where $\bar{\rho} = \frac{2}{n(n-1)}\sum_{i<j}\rho_{i,j}$ and $\rho^{☆} = \mathbb{E}^{☆}(\bar{\rho})$ and where $\mathbb{E}^{☆}$ denotes the expectation operator using probabilities that are presumed to be appropriate for correlation pricing. Note that I subtly used a whitestar symbol '☆' here instead of the blackstar symbol '⋆' to help distinguish between the correlation swap context and the variance dispersion context.

As you might expect, nobody originally had much of a clue about how to correctly price correlation, and the correlation swap strike was left to supply and demand for its determination. This resulted in correlation swaps trading at a discount to variance-based implied correlation $\rho^{\star}$: ten years ago I witnessed a gap $\rho^{\star} - \rho^{☆}$ as high as 15 correlation points (i.e., 0.15 in decimal form) on the 50 constituent stocks of the EuroStoxx index. On a $100k notional per correlation point, this *potentially* meant $1.5mn in favor of the correlation buyer—the exotics desk on the sell side.

I write "*potentially*" because, again, nobody had much of a clue about the true fair price $\rho^{☆}$ of a correlation swap versus the implied correlation $\rho^{\star}$ in a variance dispersion trade. In early 2004, I developed a simple "toy model" to price correlation swaps when the basket corresponds to a stock index, and I showed that the payoff could be replicated by dynamically trading zero-cost variance dispersions. As I suspected, my results suggested that the gap $\rho^{\star} - \rho^{☆}$ should actually be much smaller than where it was trading. The logical conclusion was that it was possible (though certainly not easy) to arbitrage correlation swaps by trading variance dispersions.

Conclusion

I hope this primer makes the topic more accessible. The problem of fair correlation swap valuation is still an open one. These days, researchers tend to favor "model-independent" approaches, but even devising a satisfactory model to price correlation is difficult. In my talk, I will present some new results on correlation fundamentals and review some correlation models.

Sébastien Bossu is currently Principal at Ogee Group LLC in New York where he runs his startup hedge fund focusing on macro option arbitrage. He is also an Adjunct Professor of Finance at Pace University. Sébastien has ten years' experience in banking and the financial industry at institutions such as J.P. Morgan, Dresdner Kleinwort and Goldman Sachs. An expert in derivative securities, he has published several papers and textbooks in the field and is a regular speaker at Global Derivatives. He is a graduate from The University of Chicago, HEC Paris, Columbia University and Université Pierre et Marie Curie.